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Exercise Set 5

Exercise 5.1. Let $\mathcal{F}$ be a laminar family of odd-cardinality subsets of $\{1, \ldots, n\}$. Show that $|\mathcal{F}| \leq \lfloor \frac{3}{2}n - \frac{1}{2} \rfloor$, and demonstrate that this is best-possible, i.e. for every n there exists a family for which the bound is tight.

(5 points)

Exercise 5.2. Let $k \in \mathbb{N}$, $k \geq 1$, and suppose G is a k-regular and $(k-1)$-edge-connected graph with an even number of vertices, and with edge weights $c : E(G) \to \mathbb{R}$. Show that there is a perfect matching M in G with
$c(M) \leq \frac{1}{k} \cdot c(E(G))$.

(5 points)

Exercise 5.3. Consider the Shortest Even/Odd Path Problem: Given a graph G with weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and $s, t \in V(G)$, find an s-t-path P of even/odd length in G that minimizes $\sum_{e \in E(P)} c(e)$ among all s-t-paths of even/odd length in G. Show that both the even and the odd version can be linearly reduced to the Minimum Weight Perfect Matching Problem.

(5 points)

Deadline: November 12^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws24/co_exercises_ws.html`

In case of any questions feel free to contact me at mkaul@uni-bonn.de.