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Exercise Set 3

Exercise 3.1. Let G be a graph, $n := |V(G)|$ even, and for any set $X \subseteq V(G)$ with $|X| \leq \frac{3}{4}n$ we have

$$\left| \bigcup_{x \in X} \Gamma(x) \right| \geq \frac{4}{3}|X|.$$

Prove that G has a perfect matching.

Hint: Let S be a set violating the Tutte condition. Prove that the number of connected components in $G - S$ with just one element is at most $\max\left\{0, \frac{4}{3}|S| - \frac{1}{3}n\right\}$. Consider the cases $|S| \geq \frac{n}{4}$ and $|S| < \frac{n}{4}$ separately.

(6 points)

Exercise 3.2. Let G be a factor-critical graph and $v, w \in V(G)$ with $v \neq w$. Prove that G contains a v-w-path P of even length and a v-w-path Q of odd length.

(4 points)

Exercise 3.3. Let G be a 2-edge-connected graph with n vertices, and let $\eta(G)$ be the minimum number of even ears in any ear-decomposition of G. Show that then for every $v \in V(G)$ there is a matching in $G - v$ of cardinality $\frac{1}{2}\Big(n - 1 - \eta(G)\Big)$.

(4 points)

Exercise 3.4. Show that a graph G is factor-critical if and only if G is connected and for every vertex $v \in V(G)$ we have $\nu(G - v) = \nu(G)$.

(4 points)

Deadline: October 29^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws24/co_exercises_ws.html`

In case of any questions feel free to contact me at mkaul@uni-bonn.de.