Exercise Set 7

Exercise 7.1. Show that the TRAVELING SALESMAN PROBLEM, restricted to instances that are the metric closure of a weighted tree, can be solved in polynomial time.

(4 points)

Exercise 7.2. Let $\lambda_{ij}, 1 \leq i, j \leq n$, be nonnegative numbers with $\lambda_{ij} = \lambda_{ji}$ and $\lambda_{ik} \geq \min\{\lambda_{ij}, \lambda_{jk}\}$ for any three distinct indices $i, j, k \in \{1, \dots, n\}$. Show that there exists a graph G with $V(G) = \{1, \dots, n\}$ and capacities $u\colon E(G) \to \mathbb{R}_+$ such that the local edge-connectivities are precisely the λ_{ij}.

Hint: Consider a maximum weight spanning tree in (K_n, c), where $c(\{i, j\}) := \lambda_{ij}$.

(5 points)

Exercise 7.3. Let G be an undirected graph and $T \subseteq V(G)$ with $|T| = 2k$ even. Prove that the minimum cardinality of a T-cut in G equals the maximum of $\min_{i=1}^{k} \lambda_{s_i,t_i}$ over all pairings $T = \{s_1, t_1, \dots, s_k, t_k\}$, where $\lambda_{s,t}$ denotes the maximum number of pairwise edge-disjoint s-t-paths.

(5 points)

Exercise 7.4. Let G be a graph, $u\colon E(G) \to \mathbb{N} \cup \{\infty\}$ and $b\colon V(G) \to \mathbb{N}$. Proof Theorem 2.29 from the lecture, i.e. show that (G, u) has a perfect b-matching if and only if for any two disjoint subsets $X, Y \subseteq V(G)$ the number of connected components C in $G - X - Y$ for which $\sum_{c \in V(C)} b(c) + \sum_{e \in E_G(v(C),Y)} u(e)$ is odd does not exceed

$$\sum_{v \in X} b(v) + \sum_{y \in Y} \left(\sum_{e \in \delta(y)} u(e) - b(y) \right) - \sum_{e \in E_G(X,Y)} u(e)$$

(6 points)

Deadline: November 28th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws19/co_exercises/exercises.html`

In case of any questions feel free to contact me at rabenstein@or.uni-bonn.de.