Exercise Set 6

Exercise 6.1. Given an undirected graph G and disjoint sets $S_e, S_o \subseteq V(G)$, a *partial* (S_e, S_o)*-join* is a set $J \subseteq E(G)$ such that $|\delta(v) \cap J|$ is even for every $v \in S_e$ and odd for every $v \in S_o$. (In particular, a T-join is the same as a partial $(V(G) \setminus T, T)$-join.) Consider the MINIMUM WEIGHT PARTIAL (S_e, S_o)-JOIN PROBLEM: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and disjoint sets $S_e, S_o \subseteq V(G)$, find a partial (S_e, S_o)-join of minimum weight, or determine that none exists. Show that this problem can be linearly reduced to the MINIMUM WEIGHT T-JOIN PROBLEM.

(4 points)

Exercise 6.2. The MINIMUM WEIGHT PERFECT SIMPLE b-MATCHING PROBLEM (MWPSb-MP) is defined as follows: Given an undirected graph G, edge-weights $c : E(G) \to \mathbb{R}$ and a function $b : V(G) \to \mathbb{N}$, find a minimum weight perfect simple b-matching in (G, c). (Note that the function b is part of the input.) Using the fact that the MINIMUM WEIGHT PERFECT MATCHING PROBLEM (MWPMP) can be solved in $O(|V(G)|^3)$-time, show that the MWPSb-MP can be solved in $O(|E(G)|^3)$-time.

(4 points)

Exercise 6.3. Consider the DIRECTED CHINESE POSTMAN PROBLEM: Given a strongly connected simple digraph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find a function $f : E(G) \to \mathbb{N} \setminus \{0\}$ such that if each edge $e \in E(G)$ is replaced by $f(e)$ copies of itself, the resulting graph is Eulerian, and such that f minimizes $\sum_{e \in E(G)} f(e)c(e)$ among functions with this property. Show that this problem can be linearly reduced to the MINIMUM COST INTEGRAL FLOW PROBLEM (i.e. the MINIMUM COST FLOW PROBLEM with the additional requirement that the flow must be integral).

(4 points)

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Exercise 6.4. An *odd cover* for a graph G is a set $F \subseteq E(G)$ such that if we successively contract in G the elements of F (and delete possible loops), the resulting graph is Eulerian. Consider the MINIMUM WEIGHT ODD COVER PROBLEM: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find an odd cover with minimum weight (or show that G has no odd cover). Show that this problem can be solved in polynomial time.

(4 points)

Deadline: November 23rd, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html`

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.