Winter term 2013/14 Research Institute for Discrete Mathematics
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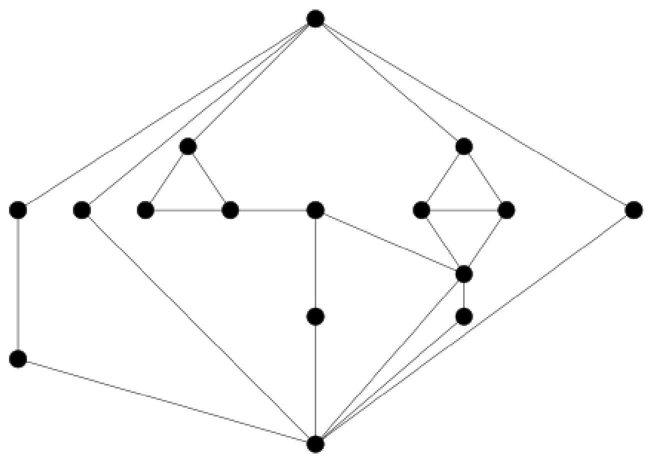
Combinatorial Optimization

Exercise Sheet 4

Exercise 4.1:
Find the Gallai-Edmonds decomposition of the following graph (and prove its correctness):



(4 points)

Exercise 4.2:
Suppose that two workers have to carry out a number of jobs. Both workers need 1 hour for each job, and there are certain jobs that need to be done before certain other jobs. The task is to get all jobs done as early as possible. This can be modeled as an acyclic directed graph $G = (V, E)$ where an edge $e = (i, j)$ means that job i has do be finished before job j is started. Let $E' := \{\{i, j\} \subseteq V \mid \text{There is neither an } i\text{-}j\text{-path nor a } j\text{-}i\text{-path in } G\}$ and set $H := (V, E')$. Prove:

- The workers cannot finish their work after less than $|V| - \nu(H)$ hours.
- The workers can finish their work after $|V| - \nu(H)$ hours.

(4 points)

Exercise 4.3:
Show that any simple graph on n vertices with minimum degree k has a matching of cardinality $min\{k, \lfloor \frac{n}{2} \rfloor\}$.

(4 points)

Deadline: Thursday, November 14, 2013, before the lecture.