Summer term 2022
Dr. U. Brenner

Research Institute for Discrete Mathematics
Universität Bonn

Linear and Integer Optimization
Assignment Sheet 11
Inofficial English Translation

1. Let $a, b \in \mathbb{R}_{>0}$ and $\{s_1, \dots, s_{37}\} \subset \mathbb{R}$. Show that for variables x, y the following constraints can be encoded in a MILP (by adding additional variables):
   (a) $(x \geq a$ or $y \geq b)$ and $x, y \geq 0$.
   (b) $x \in \{s_1, \dots, s_{37}\}$ (1+1 points)

2. Show that in the inequality
$$\max\{c^t x \mid Ax \leq bx \in \mathbb{Z}^n\} \leq \min\{b^t y \mid A^t y = c, y \geq y \in \mathbb{Z}^m\}$$
in general, equality does not hold, even if the two corresponding optimization problems are feasible and bounded. (2 points)

3. (a) Prove that a polyhedral cone is rational if and only if it is generated by a finite number of integral vectors. Conclude that $C_I = C$ for any rational cone C.
   (b) Let $P, Q \subseteq \mathbb{R}^n$ be two polyhedra. Show that $P_I + Q_I \subseteq (P + Q)_I$. Give an example where $P_I + Q_I \neq (P + Q)_I$. (3+2 points)

4. Give an example each of
   (a) a full-dimensional unbounded rational polyhedron P such that P_I is empty.
   (b) an unbounded polyhedron P such that P_I is non-empty and bounded.
   (c) a polyhedron P such that $P_I \neq \emptyset$ is not closed.
   (d) a feasible and bounded ILP without optimum solution. (1+2+2+2 points)

5. Let $P = \{x \in \mathbb{R}^{k+l} : Ax \leq b\}$ be a rational polyhedron (i.e. $A \in \mathbb{Q}^{m \times (k+l)}$, $b \in \mathbb{Q}^m$). Show that $\text{conv}(P \cap (\mathbb{Z}^k \times \mathbb{R}^l))$ is a rational polyhedron. (4 points)

Due date: Thursday, June 30, 2022, before the lecture in the lecture hall.