Summer term 2022
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Linear and Integer Optimization
Assignment Sheet 2
Inofficial English Translation

1. Let (P) be a linear program of the form $\max\{c^t x \mid Ax \leq b\}$. Show that the dual of the dual of (P) is equivalent to (P). (2 points)

2. Using the the Fourier-Motzkin elimination, decide if the following inequality systems $A_i \leq b_i$ ($i \in \{1,2\}$) have a solution. If so, specify one. If not, give a y with $y \geq 0$, $y^t A_i = 0$, and $y^t b_i < 0$.

   (a) $A_1 := \begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ -4 & 1 & -3 \end{pmatrix}$, $b_1 := \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

   (b) $A_2 := \begin{pmatrix} 1 & 5 & 0 \\ 2 & -3 & 1 \\ 0 & 5 & -1 \end{pmatrix}$, $b_2 := \begin{pmatrix} 8 \\ 2 \\ 10 \end{pmatrix}$. (2+2 points)

3. Let $A \in \mathbb{R}^{m \times (n+k)}$ and $b \in \mathbb{R}^m$. Show that

$$P = \{x \in \mathbb{R}^n \mid \exists y \in \mathbb{R}^k : A\begin{pmatrix} x \\ y \end{pmatrix} \leq b\}$$

   is a polyhedron. (4 points)

4. Let $A \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^n$ and $b, \tilde{b} \in \mathbb{R}^m$. Consider the following linear programs:

   (P1) $\max\{c^t x \mid Ax \leq b, x \geq 0\}$

   (P2) $\max\{\mathbb{1}_n^t x \mid Ax \leq b, x \geq 0\}$

   (P3) $\max\{c^t x \mid Ax \leq \tilde{b}, x \geq 0\}$

   Which of the following statements are necessarily true? Prove the correctness of your answers.

   (a) If (P1) is unbounded then (P2) is unbounded.

   (b) If (P2) is unbounded then (P1) is unbounded.

   (c) If (P1) is unbounded then (P3) is infeasible or unbounded. (2+2+2 points)

p.t.o.

5. Let $A \in \mathbb{R}^{m_1 \times n}$, $B \in \mathbb{R}^{m_2 \times n}$, $a \in \mathbb{R}^{m_1}$ and $b \in \mathbb{R}^{m_2}$. Show that the set

$$\{x \in \mathbb{R}^n \mid Ax < a, Bx \leq b\}$$

is non-empty if and only if for all $y \in \mathbb{R}^{m_1}_{\geq 0}$ and $z \in \mathbb{R}^{m_2}_{\geq 0}$ the following statements hold:

(i) If $y^t A + z^t B = 0$ then $y^t a + z^t b \geq 0$

(ii) If $y^t A + z^t B = 0$ and $y \neq 0$ then $y^t a + z^t b > 0$. (4 points)

Due date: Thursday, April 21, 2022, before the lecture in the lecture hall.

Event notice of the gender equality committee:
We invite all female, intersexual, non-binary, trans* and agender bachelor and master students to the in-person event "Tea Time With Women in Mathematics", which will take place on April 29th from 4pm (s.t.) to 6pm in the Zeichensaal, Wegelerstr. 10. You will get the chance to talk to other participants about your experiences during your studies, your plans for the future, and everything else that comes to your mind, whilst enjoying a nice cup of tea and some cookies.