Summer term 2022
Dr. U. Brenner

Research Institute for Discrete Mathematics
Universität Bonn

Linear and Integer Optimization
Assignment Sheet 1
Inofficial English Translation

1. A paper mill produces paper rolls of 3 m width. The customers order rolls with smaller widths and the mill has to cut the ordered rolls out of the 3 m wide rolls. For example, a 3 m wide roll may be cut into two 93 cm wide and a 108 cm wide roll, leaving an offcut of 6 cm. The current order consists of

   - 90 rolls of width 130 cm,
   - 610 rolls of width 108 cm,
   - 395 rolls of width 42 cm, and
   - 211 rolls of width 93 cm.

   Formulate an integer linear program that minimizes the number of produced 3 m rolls and allows a correct cutting of the ordered rolls. (5 points)

2. Let two finite disjoint sets A and B of vectors in $\mathbb{R}^2$ be given. We ask for a quadratic function $f(x) = a_2x^2 + a_1x + a_0$, such that all points in A are below the curve $\{(x, y) \mid x \in \mathbb{R}, y = f(x)\}$ and all point in B are above that curve. Describe a linear program whose solution allows you to decide directly if such a polynomial exists and, if it exists, to compute one. (5 points)

3. Show that the dimension of a non-empty set $X \subseteq \mathbb{R}^n$ is the largest d for which X contains elements $v_0, v_1, \ldots, v_d$ such that $v_1 - v_0$, $v_2 - v_0, \ldots, v_d - v_0$ are linearly independent. (3 points)

4. (a) Prove that for each set $X \subseteq \mathbb{R}^n$ the set $\text{conv}(X)$ is the smallest convex set containing X.
   (b) Prove that any set $X \subseteq \mathbb{R}^n$ with $|X| > n + 1$ can be decomposed into subsets X_1 and X_2 such that $\text{conv}(X_1) \cap \text{conv}(X_2) \neq \emptyset$. (2+5 points)

Due date: Thursday, April 14, 2022, before the lecture in the lecture hall.