

Exercise Set 11

Exercise 11.1. The METRIC BIPARTITE TRAVELING SALESMAN PROBLEM is the problem of finding a Hamiltonian cycle of minimum cost in a bipartite graph H with nonnegative cost function d satisfying

$$d(a, b) + d(b, a') + d(a', b') \geq d(a, b') \text{ for } \{a, b\}, \{a', b\}, \{a, b'\}, \{a', b'\} \in E(G).$$

Prove that for any k , if there is a k -factor approximation algorithm for the METRIC BIPARTITE TRAVELING SALESMAN PROBLEM, there is also a k -factor approximation algorithm for the METRIC TRAVELING SALESMAN PROBLEM.

(4 points)

Exercise 11.2. Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Give a $\frac{4}{3}$ -approximation algorithm for the TSP in this special case.

Hint: You may use that a minimum weight 2-matching, i.e. a minimum weight subgraph of G in which every vertex has degree 2, can be computed in polynomial time.

(4 points)

Exercise 11.3. Consider the following algorithm for the SYMMETRIC TRAVELING SALESMAN PROBLEM with triangle inequality:

Start with an arbitrary vertex $u \in V(K_n)$. Find a shortest edge $e = \{u, v\} \in E(K_n)$ connecting u to another vertex v . This yields a subtour $T = (u, v, u)$. Let $U := V(K_n) \setminus \{u, v\}$. Repeat the following steps until $U = \emptyset$:

- (i) Find $w \in U$ with shortest distance to one of the nodes in T .
- (ii) Add w to T between two neighbouring nodes $i, j \in T$ (by deleting the edge $\{i, j\}$ and connecting i and j with w), such that the cost of the new tour is minimized, i.e. find neighbouring $i, j \in T$ such that $c(i, w) + c(w, j) - c(i, j)$ is minimum. Remove w from U afterwards.

Show that this is a 2-approximation.

(4 points)

Exercise 11.4. Consider the following variant of the METRIC TSP: Given an instance of the METRIC TSP, we look for a Hamiltonian path of minimum weight (with arbitrary start- and endpoint). Give a $\frac{3}{2}$ -approximation algorithm for this problem.

(4 points)

Deadline: Tuesday, June 25th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.