Exercise Set 10

Exercise 10.1. Show that in Mehlhorn's algorithm replacing the edges of the minimum spanning tree by corresponding shortest paths does not result in cycles.

(Note: You may use that the Voronoi regions are computed with Dijkstra's algorithm.)

(4 points)

Exercise 10.2. Show that the contraction lemma still holds when the edges added between terminals have lengths larger than 0. (We add parallel edges if there already is an edge.)

(3 points)

Exercise 10.3. Show that the "vertex version" of the contraction lemma is wrong:

Construct a complete graph with metric edge lengths and vertex sets A, B and C, such that

$$0 < \mathrm{mst}(A) - \mathrm{mst}(A \cup C) < \mathrm{mst}(A \cup B) - \mathrm{mst}(A \cup B \cup C),$$

where $\mathrm{mst}(X)$ for a vertex set X denotes the length of a minimum spanning tree in the graph induced by X.

(3 points)

Exercise 10.4. Consider an instance $G = (V, E)$ of the STEINER TREE PROBLEM with terminal set R and edge length $c : E \to \mathbb{R}_+$. Denote the full components of an optimum k-Steiner tree $\mathrm{SMT}_k(R)$ with $T_1^*, \ldots, T_k^*$.

(i) Suppose that $V \setminus R$ forms a stable set. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_k^*)) .$$

(ii) Suppose that all shortest paths between any two vertices in G have length 1 or 2. Show that

$$\mathrm{mst}(R) \leq 2 \cdot (\mathrm{smt}_k(R) - \mathrm{loss}(T_1^*, \ldots, T_k^*)) .$$

(iii) Show that the Loss Contraction Algorithm achieves an approximation ratio of $1.279 \cdot r_k$ in both cases.

(2+2+2 points)

Deadline: Tuesday, June 18$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.