Prof. Dr. Stefan Hougardy
Benjamin Rockel

Exercise Set 6

Exercise 6.1. Recall the version of KNAPSACK from Exercise 5.3, where items can be used multiple times.

Give an FPTAS for this problem.

(4 points)

Exercise 6.2. Let $A = (a_i)_{1 \le i \le p}$ and $B = (b_j)_{1 \le j \le q}$ be two inputs of the BIN PACKING problem. We write $A \subseteq B$ if there are indices $1 \le k_1 < k_2 < \cdots < k_p \le q$ with $a_i \le b_{k_i}$ for $1 \le i \le p$. An algorithm for the BIN PACKING problem is called monotone if for inputs A and B with $A \subseteq B$ the algorithm needs at least as many bins for B as for A. Prove or disprove:

(i) NEXT FIT is monotone.

(i) FIRST FIT is monotone.

(2+2 points)

Exercise 6.3. Consider the MULTIPROCESSOR SCHEDULING PROBLEM: Given a finite set A of tasks, a processing time $t(a) \in \mathbb{R}_+$ for each $a \in A$ and a number m of processors, find a partition $A = \dot{\bigcup}_{i=1}^{m} A_i$ of A such that $\max_{i=1}^{m} \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.

(i) Consider a greedy algorithm that successively assigns jobs (in an arbitrary order) to the currently least used machine. Show that this is a 2-approximation algorithm.

(ii) Show that the modification of the greedy algorithm in which jobs are first sorted by $t(a)$ in non-increasing order and are then processed in that order is a $\frac{3}{2}$-approximation.

(2+2 points)

Exercise 6.4. Show that the BIN-PACKING PROBLEM restricted to instances $a_1, \ldots, a_n$ with $a_i > \frac{1}{3}$ for $i = 1, \ldots, n$ can be solved in $O(n \log n)$ time.

(4 points)

Information: The student council of mathematics will organize the math party on 9/05 in N8schicht. The presale will be held on Mon 6/05, Tue 7/05 and Wed 8/05 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de.

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Deadline: Thursday, May 16$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.