# Exercise Set 3

**Exercise 3.1.** Consider the following variant of the $k$-Center problem:

**Instance:** A complete graph $G = (V, E)$, metric edge weights $d : E(G) \to \mathbb{R}$, a partition $V = C \dot\cup S$, an integer $k \in \mathbb{N}$.

**Output:** A set $X \subseteq S$ with $|X| \leq k$ that minimizes

$$\max_{c \in C} \left\{ \min_{s \in X} \{d(c, s)\} \right\}.$$

(i) Show that this problem does not admit a $(3-\varepsilon)$-approximation for any $\varepsilon > 0$ unless P=NP.

(ii) Give a 3-approximation algorithm.

(4+4 points)

**Exercise 3.2.** Show that, unless P=NP, for any $\alpha$ polynomial in the input size, there is no $\alpha$-approximation algorithm for the $k$-Center problem if we do not require the distance function to satisfy the triangle inequality.

(2 points)

**Exercise 3.3.** Consider the Directed Steiner Tree Problem: Given an edge-weighted digraph $G = (V, E)$, a set of terminals $T \subseteq V$ and a root vertex $r \in V$, find a minimum weight arborescence rooted at $r$ that contains every vertex in $T$.

Show that a $k$-approximation algorithm for the Directed Steiner Tree Problem can be used to obtain a $k$-approximation algorithm for Minimum Weight Set Cover.

(2 points)

**Exercise 3.4.** An instance of Max-Sat is called $k$-satisfiable if any $k$ of its clauses can be satisfied simultaneously. Give a polynomial-time algorithm that computes for every 2-satisfiable instance a truth assignment which satisfies at least a $\frac{\sqrt{5}-1}{2}$-fraction of the clauses.

*Hint:* Some variables occur in one-element clauses (w.l.o.g. all one-element clauses are positive), set them *true* with probability $a$ (for some constant $a \in [0, 1]$), and set the other variables *true* with probability $\frac{1}{2}$. Choose $a$ appropriately and derandomize this algorithm.

(4 points)

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**Deadline:** Thursday, April 25$^{th}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.