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Exercise Set 5

Exercise 5.1. Let $T \subset \mathbb{R}^2$ be a finite set of terminals, and $S_1, \dots, S_m \subseteq \mathbb{R}^2$ be rectangular, axis-parallel blockages. Let $S := \bigcup_i S_i$, $\mathring{S}$ denote the interior of S, and let $0 < L \in \mathbb{R}$ be a constant.

A rectilinear Steiner tree Y for T is *reach-aware* if every connected component of $E(Y) \cap \mathring{S}$ has length at most L.

(a) We define the *Hanan grid induced by* $(T, S_1, \dots, S_m)$ as the usual Hanan grid for $T \cup \{l_i, u_i \mid 1 \leq i \leq m\}$ where l_i (resp. u_i) is the lower left (resp. upper right) corner of S_i.

Prove or disprove: There is always a shortest reach-aware Steiner tree for T that is a subgraph of the Hanan grid induced by $(T, S_1, \dots, S_m)$.

(b) Prove that it is $\mathcal{NP}$-hard to compute a reach-aware Steiner tree for T that has at most twice the length of an optimum solution.

Hint: If there are no terminals on blockages this is not $\mathcal{NP}$-hard.

(2+2 points)

Exercise 5.2. Consider the following problem:

> RSMT WITH WEIGHTED SUM OF ELMORE DELAYS
>
> **Input:** A finite set $T \subset \mathbb{R}^2$, a root $r \in T$; source resistance R_0, resistance R_u and capacitance C_u per wire unit; weights $w_i > 0$ and downstream capacitance downcap(t_i) for each $t_i \in T \setminus \{r\}$.
>
> **Task:** Construct a rectilinear Steiner tree Y for T that minimizes $\sum_{t_i \in T \setminus \{r\}} w_i \textsc{Elmore}(r, t_i)$.

For a Steiner tree Y for T and $t_i \in T \setminus \{r\}$ we define $\textsc{Elmore}(r, t_i)$ as

$$\textsc{Elmore}(r, t_i) := R_0 \,\text{downcap}(r) + \sum_{e=(u,v) \in Y[r,t_i]} \text{res}(e) \left(\frac{\text{cap}(e)}{2} + \text{downcap}(v) \right)$$

where $Y[r, t_i]$ denotes the $r - t_i$-path in Y and $\text{cap}(e) := C_u\, \ell_1(u, v)$ and $\text{res}(e) := R_u\, \ell_1(u, v)$. For $v \in V(Y) \setminus (T \setminus \{r\})$ we define

$$\text{downcap}(v) := \sum_{e=(v,w)\in\delta^+_Y(v)} (\text{cap}(e) + \text{downcap}(w))$$

(a) Prove that the Hanan grid for T contains an optimum solution to RSMT with Weighted Sum of Elmore Delays.

(b) Show that this does not hold if, instead of the weighted sum, we minimize $\max_{t_i \in T\setminus\{r\}} \text{Elmore}(r, t_i)$.

(4+4 points)

Exercise 5.3. Consider the following Clustered Rectilinear Steiner Tree Problem: Given a partition $T = \dot{\bigcup}_{i=1}^k P_i$ of the terminals ($\emptyset \neq P_i \subseteq \mathbb{R}^2, |P_i| < \infty$), find a (rectilinear) Steiner tree Y_i for each set of terminals P_i and one rectilinear, toplevel (group) Steiner tree Y_{top} connecting the embedded trees Y_i ($i = 1, \ldots, k$). The task is to minimize the total length of all trees.

Let A be an α-approximation algorithm for the Rectilinear Steiner Tree Problem. A feasible solution to the Clustered Rectilinear Steiner Tree Problem can be found by first selecting a connection point $q_i \in \mathbb{R}^2$ for each $i = 1, \ldots, k$ and then computing $Y_i := A(P_i \cup \{q_i\})$ and $Y_{\text{top}} := A(\{q_i : 1 \leq i \leq n\})$.

(a) Show that picking $q_i \in P_i$ arbitrarily yields a 2α approximation.

(b) Prove that choosing each q_i as the center of the bounding box of P_i implies a $\frac{7}{4}\alpha$ approximation algorithm.

(c) Show that both approximation ratios above are tight.

(2 + 4 + 2 points)

Deadline: May 29th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/chipss18.html`

In case of any questions feel free to contact me at bihler@or.uni-bonn.de.