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Exercise Set 10

Exercise 10.1. For a directed graph (V, E) with non-negative edge weights c that fulfill the triangle inequality, consider the subtour LP for the asymmetric TSP:

$$\begin{array}{llll} \min & c(x) & & \\ \text{s.t.} & x(\delta^+(U)) & \geq & 1 & (\emptyset \neq U \subset V) \\ & x(\delta^+(v)) & = & x(\delta^-(v)) & (v \in V) \\ & x_e & \geq & 0 & (e \in E). \end{array} \tag{1}$$

Prove that for any $n \geq 3$, the integrality ratio of the subtour LP for the asymmetric TSP (1) is the smallest number ρ such that for every feasible solution x^* of (1) the vector ρx^* is a convex combination of integral solutions to (1).

(5 points)

Exercise 10.2. Consider the following algorithm for the Asymmetric TSP with triangle inequality: Given a complete graph G on n vertices and $c : E(G) \to \mathbb{R}_+$, find a directed cycle C in G minimizing $\frac{c(E(C))}{|C|}$ and add the edges of C to the solution. Remove all but one of the vertices of C from G and proceed recursively until G is reduced to a single vertex.

Prove that this is a $(2 \ln n)$-approximation algorithm for the ATSP.

(You may use that the cycle C can be found in polynomial time.)

(4 points)

Exercise 10.3. Prove that the integrality gap of the subtour elimination LP for the s-t-path TSP is at most $\frac{5}{3}$.

(5 points)

Exercise 10.4. Let x be a feasible solution to the subtour elimination LP for the s-t-path TSP.

(a) Prove that there exist sets $X_1, \ldots, X_m$ with

$$\{s\} \subseteq X_1 \subset X_2 \subset \cdots \subset X_m \subseteq V \setminus \{t\}$$

such that

$$\{\delta(X_i) : i \in \{1, \ldots, m\}\} = \{\delta(U) : \emptyset \neq U \subset V,\, x(\delta(U)) < 2\}.$$

(b) Prove that there exists a spanning tree S in $(V, \{e \in E : x_e > 0\})$ such that $|S \cap \delta(X_i)| = 1$ for $i = 1, \ldots, m$.

(2+4 points)

Deadline: Tuesday, July 10^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/appr_ss18_ex.html`

In case of any questions feel free to contact me at traub@or.uni-bonn.de.