Exercise Set 4

Exercise 4.1. Let $A = (a_i)_{1 \le i \le p}$ and $B = (b_j)_{1 \le j \le q}$ be two inputs of the Bin Packing problem. We write $A \subseteq B$ if there are indices $1 \le k_1 < k_2 < \cdots < k_p \le q$ with $a_i \le b_{k_i}$ for $1 \le i \le p$. An algorithm for the Bin Packing problem is called monotone if for inputs A and B with $A \subseteq B$ the algorithm needs at least as many bins for B as for A. Prove or disprove:

(a) Next Fit is monotone.

(b) First Fit is monotone.

(4 points)

Exercise 4.2. Consider the Multiprocessor Scheduling Problem: Given a finite set A of tasks, a processing time $t(a) \in \mathbb{R}_+$ for each $a \in A$ and a number m of processors, find a partition $A = \dot{\bigcup}_{i=1}^{m} A_i$ of A such that $\max_{i=1}^{m} \left\{ \sum_{a \in A_i} t(a) \right\}$ is minimum.

(a) Consider a greedy algorithm that successively assigns jobs (in an arbitrary order) to the currently least used machine. Show that this is a 2-approximation algorithm.

(b) Show that the modification of the greedy algorithm in which jobs are first sorted by $t(a)$ in non-increasing order and are then processed in that order is a $\frac{3}{2}$-approximation.

(4 points)

Exercise 4.3. Give an algorithm for Bin Packing restricted to instances with a constant number of different item sizes whose running time is polynomially bounded in the number n of items.

Hint: Use dynamic programming.

(4 points)

Exercise 4.4. For an instance of Bin Packing, let $(s_1, \ldots, s_m)$ denote the different item sizes and $(b_1, \ldots, b_m)$ their multiplicities. Denote by

$$\{T_1, \ldots, T_N\} = \left\{ (k_1, \ldots, k_m) \in \mathbb{Z}_+^m : \sum_{i=1}^{m} k_i s_i \le 1 \right\}$$

all possible configurations for a single bin, where $T_j = (t_{j1}, \dots, t_{jm})$ for $j = 1, \dots, N$. Consider the following LP

$$\begin{aligned} \min \quad & \sum_{j=1}^{N} x_j \\ s.t. \quad & \sum_{j=1}^{N} t_{ji} x_j \geq b_i & (i = 1, \dots, m) \\ & x_j \geq 0 & (j = 1, \dots, N) \end{aligned}$$

Let LP denote the optimum value of this LP and let OPT denote the value of an optimum integral solution (i.e. an optimum solution to the Bin Packing problem).

Show that there exists an instance of Bin Packing whith $\lceil \text{LP} \rceil < \text{OPT}$.

(4 points)

Exercise 4.5. Show that any bin packing instance with only m different item sizes has an optimum solution with at most 2^m different bin configurations.

Hint: For a solution with more bin configurations, there are two bins for which the quantities of packed items have the same parities.

(4 points)

Deadline: Tuesday, May 15$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/appr_ss18_ex.html`

In case of any questions feel free to contact me at traub@or.uni-bonn.de.