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Exercise Set 10

Exercise 10.1. Let (G, H) be a pair of undirected graphs on $V(G) = V(H)$ with capacities $u : E(G) \to \mathbb{R}_+$ and demands $b : E(H) \to \mathbb{R}_+$. A *concurrent flow* of value $\alpha > 0$ is a family $(x^f)_{f \in E(H)}$ where x^f is an s-t-flow of value $\alpha \cdot b(f)$ in $(V(G), \{(v,w),(w,v) \mid \{v,w\} \in E(G)\})$ for each $f = \{t, s\} \in E(H)$, and

$$\sum_{f \in E(H)} x^f\Big((v,w)\Big) + x^f\Big((w,v)\Big) \leq u(e)$$

for all $e = \{v, w\} \in E(G)$. The Maximum Concurrent Flow Problem is to find a concurrent flow with maximum value $\alpha > 0$.

Prove that the Maximum Concurrent Flow Problem is a special case of the Min-Max Resource Sharing Problem. Specify how to implement block solvers.

(5 points)

Exercise 10.2. Consider the Escape Routing Problem: We are given a complete 2-dimensional grid graph $G = (V, E)$ (i.e. $V = \{0, \dots, k-1\} \times \{0, \dots, k-1\}$ and $E = \{\{v, w\} \mid v, w \in V, \|v - w\| = 1\}$) and a set $P = \{p_1, \dots, p_m\} \subseteq V$. The task is to compute vertex-disjoint paths $\{q_1, \dots, q_m\}$ s.t. each q_i connects p_i with a point on the border $B = \{(x, y) \in V \mid \{x, y\} \cap \{0, k-1\} \neq \emptyset\}$.

Find a polynomial-time algorithm for the Escape Routing Problem or prove that the problem is NP-hard.

(4 points)

Exercise 10.3. Show that the Vertex-Disjoint Paths Problem is NP-complete even if G is a subgraph of a track graph G_T with two routing planes. Recall that in this case G_T is a graph $G_T = (V, E)$ for some $n_x, n_y \in \mathbb{N}$ with $V = \{1, \dots, n_x\} \times \{1, \dots, n_y\} \times \{1, 2\}$ and $E = \{\{(x, y, z), (x', y', z')\} : |x - x'|z + |y - y'|(3 - z) + |z - z'| = 1\}$.

Hint: Consider the proof of Theorem 6.2.

(5 points)

Exercise 10.4. Given an instance of the Min-Max Resource Sharing Problem with σ-optimal block solvers for some fixed $\sigma \geq 1$.

(a) Show that t phases of the Resource Sharing Algorithm call the oracle at most

$$\min\left\{t\Lambda,\ t|\mathcal{N}| + \frac{|\mathcal{R}'|}{\varepsilon}\ln\left(\mathbb{1}^\top y^{(t)}\right)\right\}$$

times where $\Lambda := \sum_{N\in\mathcal{N}} \max\{1, \sup\{b_r \mid r \in \mathcal{R}, b \in \mathcal{B}_N\}\}$ and $\mathcal{R}' := \{r \in \mathcal{R} \mid \exists N \in \mathcal{N}, b \in \mathcal{B}_N \text{ with } b_r > 1\}$.

(b) Prove that a $\sigma(1+\omega)$-approximate solution can be computed in

$$O\Big(\theta \log|\mathcal{R}| \left(\left(|\mathcal{N}| + |\mathcal{R}|\right) \log\log|\mathcal{R}| + \sigma\omega^{-2} \min\left\{\rho|\mathcal{N}|, |\mathcal{N}| + |\overline{\mathcal{R}}|\sigma\right\}\right)\Big)$$

time where $\rho := \max\{1, \sup\{b_r/\lambda^* \mid r \in \mathcal{R}, N \in \mathcal{N}, b \in \mathcal{B}_N\}\}$ and $\overline{\mathcal{R}} := \{r \in \mathcal{R} \mid \exists N \in \mathcal{N}, b \in \mathcal{B}_N \text{ with } b_r > \lambda^*\}$.

Remark: For practical routing instances ρ and $|\overline{\mathcal{R}}|$ are usually small.

(2 + 4 points)

Deadline: July 11th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss17/chipss17.html`

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.