Exercise Set 5

Exercise 5.1. Let $T \subset \mathbb{R}^2$ be a finite set of terminals, and $S_1, \dots, S_m \subseteq \mathbb{R}^2$ be rectangular, axis-parallel blockages. Let $S := \bigcup_i S_i$, $\mathring{S}$ denote the interior of S, and let $0 < L \in \mathbb{R}$ be a constant.

A rectilinear Steiner tree Y for T is *reach-aware* if every connected component of $E(Y) \cap \mathring{S}$ has length at most L. We define the *Hanan grid induced by* $(T, S_1, \dots, S_m)$ as the usual Hanan grid for $T \cup \{l_i, u_i \mid 1 \leq i \leq m\}$ where l_i (resp. u_i) is the lower left (resp. upper right) corner of S_i.

Prove or disprove: There is always a shortest reach-aware Steiner tree for T that is a subgraph of the Hanan grid induced by $(T, S_1, \dots, S_m)$.

(4 points)

Exercise 5.2. Let (G, c, T) be an instance of the Steiner Tree Problem, G connected, $t \in T$ a terminal and $k \in \mathbb{N}$ with $k \geq 1$.

(a) For two valid lower bounds lb_a and lb_b, define $\max(\mathrm{lb}_a, \mathrm{lb}_b)$ by

$$\max(\mathrm{lb}_a, \mathrm{lb}_b)(v, I) := \max\Big(\mathrm{lb}_a(v, I), \mathrm{lb}_b(v, I)\Big).$$

Show that $\max(\mathrm{lb}_a, \mathrm{lb}_b)$ also defines a valid lower bound.

(b) Prove that $\mathrm{lb}_{\mathrm{BB}}(v, I) := \mathrm{BB}(\{v\} \cup I)$ is a valid lower bound for instances of the Rectilinear Steiner Tree Problem.

(c) Show that $\mathrm{lb}_{\mathrm{mst}}(v, I) := \frac{\mathrm{mst}(\{v\} \cup I)}{2}$ defines a valid lower bound.

(d) Define $\mathrm{lb}_k(v, I) := \max\Big\{\mathrm{smt}(J) \;\Big|\; t \in J \subseteq I \cup \{v\}, |J| \leq k+1\Big\}$ if $t \in I$ and $\mathrm{lb}_k(v, I) := 0$ otherwise. Show that lb_k is a valid lower bound.

(3 + 1 + 1 + 4 points)

Exercise 5.3. Consider the following Clustered Rectilinear Steiner Tree Problem: Given a partition $T = \dot{\bigcup}_{i=1}^{k} P_i$ of the terminals ($\emptyset \neq P_i \subseteq \mathbb{R}^2, |P_i| < \infty$), find a (rectilinear) Steiner tree Y_i for each set of terminals P_i and one rectilinear, toplevel (group) Steiner tree Y_{top} connecting the embedded trees Y_i ($i = 1, \ldots, k$). The task is to minimize the total length of all trees.

Let A be an α-approximation algorithm for the Rectilinear Steiner Tree Problem. A feasible solution to the Clustered Rectilinear Steiner Tree Problem can be found by first selecting a connection point $q_i \in \mathbb{R}^2$ for each $i = 1, \ldots, k$ and then computing $Y_i := A(P_i \cup \{q_i\})$ and $Y_{\text{top}} := A(\{q_i : 1 \leq i \leq n\})$.

(a) Show that picking $q_i \in P_i$ arbitrarily yields a 2α approximation.

(b) Prove that choosing each q_i as the center of the bounding box of P_i implies a $\frac{7}{4}\alpha$ approximation algorithm.

(c) Show that both approximation ratios above are tight.

(2 + 4 + 2 points)

Deadline: May 30th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss17/chipss17.html`

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.